How Evolution Created God: The Search for the Origins of Religion

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Abstract

The easiest way of understanding religion is through the eyes of a believer. However, a non-believer often cannot accept this vision because it refers to beings and forces that he or she cannot independently verify. Religion is even more of a puzzle to the scientist, who is taught to believe only in a reality that can be observed and tested. Yet, religion exists in the minds and hearts of people. How did it get there if it was not implanted by a divine being? Natural selection is the only process that science knows of that can produce a brain that experiences religion. This process by which natural selection has produced the religious brain is still unclear. One way of discovering it is through mathematical and agent-based simulation. This paper will present an agent-based model for the natural selection of unreal beliefs and discuss further mathematical approaches to the problem.

Introduction

This paper is purely exploratory. It is a reconsideration of an old problem in cultural anthropology, the origin of religion. It does not produce the final word on the origin; instead, it examines some previous work on the subject and it develops some conceptual tools that will help to solve the problem in the future.

As a science, cultural anthropology produces two things, data and theory. Of course, some cultural anthropologists would say that cultural anthropology is not a science at all and that it should not concern itself with scientific forms of knowledge (Marcus and Fischer 1986); however, there are others who believe that scientific knowledge can grow within the field of cultural anthropology (Kuznar 1997). I have taken this latter point of view. Data in cultural anthropology is abundant, although it is often presented in heaps of text, photos, or diagrams that are difficult to organize. For this reason and the fact that human behavior is a terribly complex phenomenon, good cultural anthropological theories have progressed very slowly.

When anthropology first appeared, it was hoped that its new scientific perspective would develop some good theories of human behavior. One of the earliest questions asked of scientific anthropology was: where did religion come from? It was phrased in evolutionary terms as the search for the origin of religion. In the nineteenth century, Edward Burnett Tylor (1960) tackled the problem and developed a psychological theory to explain the origin. He saw it in dream experiences that could most easily be explained as visions of an unseen alternate world. A vast literature developed around this topic, but a clear, empirically testable theory did not emerge. Fifty years later, Émil Durkheim (1912) turned away from the evolutionary perspective that had held sway for most of the 19th century and developed a sociological theory that was very vague about any temporal sequence. He decided that religion was created by "society" seen in a rather mystical way. Later in the early 20th century, Malinowski (1948:36), a prolific anthropologist, decided that religion could not even be clearly defined. The question of religion's origin became a rather moribund issue throughout the twentieth century as scientific anthropology turned toward gathering more data. Thus, one of the earliest questions posed to scientific anthropology was never answered.

The early anthropologists, such as Tylor, saw the problem within a framework of cultural progress. They were convinced that cultures were progressing from less complicated forms to more complex ones. Thus, the origin of religion was to be found among the simpler cultures, which they called "primitive" and thought of as hold-backs in march of "progress." As time went on, the search for the origin gradually broke free from this tangled web of nineteenth century cultural progressivism. Now the question has reemerged as a question about evolution of religious behavior. It is now a question about the biological evolution of the human brain and the evolution of cultural cognition. It is no longer a question about cultural "progress."

Some modern theories that I call *cognitivist* look at the evolution of religious ideas rather than at the evolution of religious behavior. Boyer (1994), for example, searches for the cognitive styles found in religion. There is no trouble in finding them. When they are found, their functions almost always lie outside of religion, and the cognitivists conclude that religion is a collection of cognitive styles that have evolved for other purposes. However, one can still ask why do these styles consistently come together in religious behavior? Could not religion be adaptive as a special complex of cognitive styles and behavior that works as a special human behavioral system? Most people react to it as such a behavioral system. So the cognitivists go against common sense, but science is allowed to do that.

Other theorists (*eg*. Sosis 2004, Alcorta and Sosis 2005, Bulbulia et al. 2008) see religion evolving as a behavioral system. Religious behaviors are so universal that they must have some adaptive value on their own. Religious people send out signals that make other people trust them. They strengthen the social matrix as Durkheim observed. The problem for these *commitment* theorists is that they don't know how the system of religious behavior was selected by natural selection, although they can see the adaptive functions in it. Animal models of costly signaling (Zahavi 1975) only provide distant analogs. Understanding the natural selection of human behavioral complexes is very difficult when they are not directed toward an obvious survival or reproductive goal such as food production. The thrust of religion, to worship unseen beings whose existence is unverifiable, seems to have no obvious fitness benefit. In fact, it seems downright destructive. An animal that does not create an accurate internal representation of its external environment seem less likely to survive than one that does.

One clue to the selection of religious behavior has been clear since the time of Durkheim. Religion aids successful social organization beyond the range of kinship. The selection of religious behavior may have been due to the way it organizes successful cooperation. Durkheim (1912) saw this but did not have the analytical tools to unravel the process. Instead, he went on to study "society," his reification of the essence of social organization, which he felt was rooted in a religious impulse.

Social selection is any selection process in which individuals respond to each other's behavior, or signals. They give or withhold reproductive benefits in a way that depends on characteristics that they see in other individuals. Some of the results of social selection, such as reciprocity, seem almost obvious. Others, religion for example, are completely mysterious, not offering any obvious clues to their adaptive potential. Social selection adapts a species to live within groups of members of the same species. A type of social selection was the topic of one of Darwin's (1871) later books, *The Descent of Man, and Selection in Relation to Sex*.[1] He was puzzled by the evolution of traits such as the peacock's tail and the deer's antlers that seemed to be more handicaps than benefits in the struggle for survival. These features were selected by members of the same species, namely the females, who were trying to improve the survival of their genes by selecting good mates. The males evolved the features that helped them compete for mates. Social selection is any selection process carried out by members of the same species who have the ability to affect the reproduction of others.

Modern researchers are now offering and testing new hypotheses that relate to the selection of religious behavior. Sociologists (Stark and Fink 2000; Iannaccone 1998) have revealed an adaptive rationality to religious choices. People tend to choose religions that benefit themselves. Irons (2001) and Sosis (2003) have connected religion to a process of costly signaling. Religious people engender trust because they are generous and not concerned with themselves. By paying the costs of commitment, the religious believer shows that he or she can be trusted. This trust, in turn, creates an advantageous network of cooperation

[1]The *The Descent of Man, and Selection in Relation to Sex* was published in 1871, twelve years after his famous treatise *On the Origin of Species by Means of Natural Selection, or the Preservation of Favored Races in the Struggle for Life.*

that has fitness advantages for the signaler. Other people give benefits to religious people because they believe they can trust them. Bulbulia (2004) nails the process of selection down to an advantage in playing cooperative games such as the Prisoner's Dilemma. The Prisoners Dilemma is often used as a paradigm for the problem of human cooperation. Two persons are placed in a situation where one can defeat the other by acting selfishly and where they both can get more by cooperating. Which solution is taken depends on the trust that exists between the two players.[2] Religion overcomes a tendency toward selfish defection that can hurt both parties. Thus, the social sciences are inching toward a solution of a primary theoretical question, the origin of religion; however, there still is a long way to go.

Recently I (Dow 2008) published an article that shows that natural social selection can account for the evolution of religion. The argument is based on a computer simulation of abstract agents who believe in non-verifiable entities. The computer simulation was elementary and did not try to model real persons. It simply showed one way in which religious beliefs, and religion itself, could be selected as a system by natural selection. The major conclusion was that a belief in unverifiable entities could evolve if it had a greenbeard effect. The term greenbeard refers to Hamilton's and Dawkins' (1976) evolutionary hypothesis that a genetically determined signal can allow selfish genes to produce altruistic behavior. He chose an arbitrary genetically determined signal, that of a green beard, to illustrate the phenomena. The proclamation of belief in supernatural beings could be a greenbeard signal to others. If others respond to the signal with benefits, it can maintain a population of believers.

Evogod: The Simulation Project

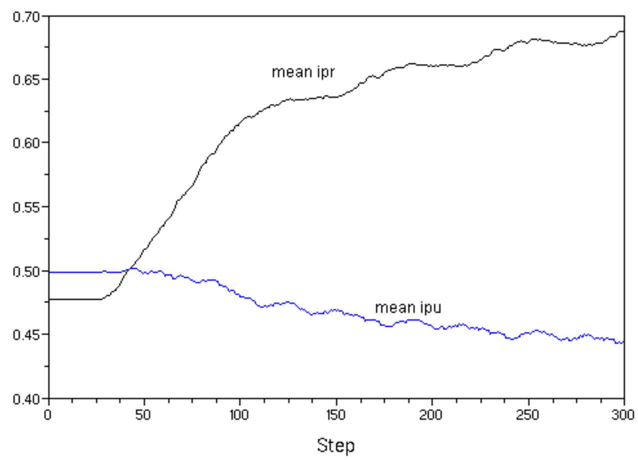
In the simulation to which I referred, a society of agents provides a framework in which a genetically transmitted capacity for the belief in non-verifiable entities can evolve. The agent-based model is labeled *evogod* for convenience (Dow 2008). It is a co-evolutionary model, one that includes both genetically determined and learned behavior.[3] The genetic factor is a varying capacity to learn behavior.

The agents are asexual reproducers. Each agent is capable of two types of communication, real communication and unreal communication. Real communication carries information about the environment that the receiver can use to increase its fitness. Unreal communication carries no information about the environment and might decrease the fitness of the receiver by diverting its energies. Religious communication is of the unreal type. An agent learns to communicate by receiving communications from other agents. Each agent carries a fixed capacity to learn real communication and another capacity to learn unreal communication. A full range of capacities is assigned to the initial generation of agents. Eventually each agent dies at a fixed age, and its capacities

[2]See Axelrod (1984) and Cohen (1999).

[3]For a general discussion of the combined evolution of genes and culture see Durham (1991).

Figure 1: No greenbeard effect on the evolution of mean capacity for realistic (ipr) and religious (ipu) communication

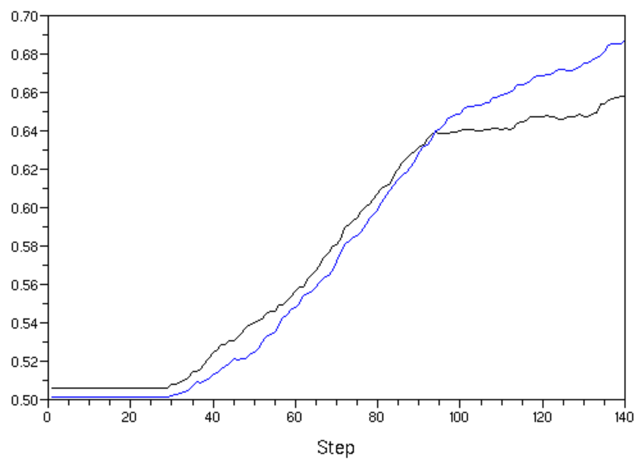


are inherited by "offspring," who are inserted into the population without any tendency to communicate, but with the parent's capacities to learn either type of behavior. A parent who has accumulated more fitness produces more offspring. The mean value of these capacities in the population changes as agents die and new ones are "born."

During each step of the model, an agent communicates, really and unreally, with a fixed number of other agents. The other agents with whom an agent communicates can be chosen randomly from a uniform distribution of the agents (greenbeard = 0), or they can be chosen from a distribution based on the tendency of another agent to engage in unreal communication (greenbeard = 1). The greenbeard parameter models the ability of agents to respond to the behavior of other agents. When the greenbeard parameter is turned on (greenbeard = 1), agents select other agents as recipients of communication on the basis of the other agent's ability to communicate unreal information. The greenbeard effect models Iron's (2001) and Sosis's (2004) hard-to-fake signals. In their theory, religious communicators evoke greater trust causing others to desire an association with them. The costliness of a religious signal in their theory is included in the *evogod* model as the parameter *dfiu*.

The simulation shows that a capacity for religious communication will not evolve simply as a stimulus to general communication. Figure 1 shows the capacity for real (non-religious) and unreal (religious) communication evolving when there is no greenbeard effect. The mean capacity for real communication increases, and the mean capacity for unreal communication goes down. However, when the greenbeard effect is turned on, the capacity for unreal communication increases as seen in Figure 2.

Figure 2: The greenbeard effect on the evolution of mean capacity for realistic (black) and religious (blue) communication



Tracing the Origins of Religion

At this point in the quest for the origins of religion, the path becomes difficult. Solutions are seen only in a vague outline. They need to be hypothesized and tested. I will move on by discussing a tool that can be used to formulate testable hypotheses. It is a revision of basic evolutionary theory to make it simpler and more applicable to social selection. The scheme shows that the problem is not easily solved with simple estimates based in linear assumptions. Multiagent simulation, which has been moving ahead in the area of theoretical social psychology and computer science, may be needed to advance understanding to a level where mathematical analysis can again pick it up.

Revisions of Basic Evolutionary Theory

Religious behavior is most certainly a polygenic capacity. No one has yet found a particular gene that produces a religious part of the human brain. On the contrary, the behaviors that make up religion are numerous and are mixed together in varying proportions. Therefore, a continuous variable model appropriate to polygenic traits seems more appropriate than a discrete model that is often used in single-gene and game theory. The techniques of mathematical statistics that make use of Lebesgue measure and Lebesgue integration (Cramér 1945) can be used for polygenic models. If the equations formulated with Lebesgue integration are too difficult solve, the variables can be expressed in discrete form for simpler numerical testing of particular cases. Lebesgue integrals become sums when the variables are discrete. This is the great advantage of Lebesgue measure, it can make use of either continuous or discrete probability variables.

Most mathematical evolutionary theory has employed discrete variates to describe specific alleles or game strategies (Maynard Smith 1982, McElreath and Boyd 2007). This has produced simple models that have often led to insights about more complex processes. The discrete behavioral models have been particularly valuable in understanding the stable equilibria that have resulted from a conflict of interests (Maynard Smith 1982); however, while looking at the evolution of religion, it seems appropriate to begin with the idea that the capacity to communicate unverifiable images of an external reality is inherited by degrees and not just present or absent. Thus, treating it as a continuous variable is more appropriate. Lebesgue measure offers the advantage of expressing statistical relationships in such a way that the theory applies both to complex polygenic traits and simple discrete ones.

The general evolutionary model

A trait is measured by a variable z, discrete or continuous. A fitness function $b(z)$ exists for each value of the trait. $b(z)$ affects the reproduction of the trait z in the next generation. Several assumptions can be made to simplify a model. (1) The members of the population, the carriers of the trait, can be modeled as haploid reproducers if one does not want to consider varying effects of sex on the genetic transmission. (2) Changes in the environment can be left out of the model in order to explore the underlying social mechanisms. (3) The possibility of mutations can be ignored in order to explore the implications of social systems on the biological evolution of behavior. These assumptions are typical of models of the evolution of social behavior.

Evolution can be described as a transform of probability distributions. The various values of a trait z are distributed with the probability $f(z)$ in a parental generation. A single step in the evolutionary process takes place when the fitness function, $b(z)$, associated with each value of z transforms the distribution $f(z)$ in the parent generation into a new distribution $f'(z)$ in the child generation. Let $b(z)$ be defined by the number of offspring produced by an individual with the trait z in the parent generation. One can think of $b(z)$ as the number of type-z individuals who will be reproduced in the next generation. $b(z)$ must be greater than or equal to zero. One cannot produce negative offspring. If $b(z)$ is greater than one, it represents a reproductive benefit that produces more individuals of type z in the next generation. If it is less than one but greater than zero, it represents a reproductive punishment. Fewer type-z individuals will be produced. Punishments as well as benefits are considered in this model.

m individuals carrying the trait z in the parent generation will produce $m\,b(z)\,f(z)$ individuals in the next generation. The total number of offspring of type z in the next generation will be

$$\int_{-\infty}^{\infty} m\,b(z)\,f(z)\,dz$$

so the frequency of individuals with the trait z in the next generation is

$$f'(z) = \frac{m\, b(z)\, f(z)}{\int_{-\infty}^{\infty} m\, b(z)\, f(z)\; dz} = \frac{b(z)\, f(z)}{\int_{-\infty}^{\infty} b(z)\, f(z)\; dz} \tag{1}$$

This can also be expressed as a recursion equation from generation n to generation $n+1$.

$$f_{n+1}(z) = \frac{b(z)\, f_n(z)}{\int_{-\infty}^{\infty} b(z)\, f_n(z)\; dz}$$

This multiplicative transform model is a typical way of looking at the mathematics of evolution. I call it multiplicative because $b(z)$ multiplies the individuals of type z in the next generation. Typically, population genetics has been concerned with the change of average values of quantitative traits. The classical Price equation can be derived from Equation 1 as follows. If z is a quantitative trait, for example a tendency toward generosity measured by the probability of giving a gift, then the mean of z in the parent population is $\int_{-\infty}^{\infty} z\, f(z)\, dz$. The change in the mean value of z from one generation to the next is

$$\Delta\bar{z} = \int\limits_{-\infty}^{\infty} z\, f'(z)\, dz - \int\limits_{-\infty}^{\infty} z\, f(z)\, dz$$

Substituting the values from equation 1 gives

$$\Delta\bar{z} = \frac{\int_{-\infty}^{\infty} z\, f(z)\, b(z)\, dz}{\int_{-\infty}^{\infty} f(z)\, b(z)\, dz} - \int\limits_{-\infty}^{\infty} z\, f(z)\, dz \tag{2}$$

Multiplying both sides of Equation 2 by $\int_{-\infty}^{\infty} f(z)\, b(z)\, dz$ gives

$$\Delta\bar{z} \int\limits_{-\infty}^{\infty} f(z)\, b(z)\, dz = \int\limits_{-\infty}^{\infty} z\, f(z)\, b(z)\, dz - \int\limits_{-\infty}^{\infty} z\, f(z) \int\limits_{-\infty}^{\infty} f(z)\, b(z)\, dz \tag{3}$$

$\int_{-\infty}^{\infty} f(z)\, b(z)\, dz$ can be called the mean fitness of z in the parent generation and written as

$$\bar{b} = \int\limits_{-\infty}^{\infty} f(z)\, b(z)\, dz$$

Thus Equation 3 becomes

$$\bar{b}\,\Delta\bar{z} = \int\limits_{-\infty}^{\infty} z\, f(z)\, b(z)\, dz - \bar{b}\,\bar{z} \tag{4}$$

The right hand side of this equation is a formula for covariance so the above can be rewritten as

$$\bar{b}\,\Delta\bar{z} = \mathrm{Cov}(b, z) \tag{5}$$

This is the Price equation without the term for changes in the environment and without a term for mutation Frank(1998:14). . A discussion in the Appendix shows why the right hand side of Equation 4 is the covariance between b and z. The Price Equation was independently discovered by Robertson (1966), Li (1967), and Price (1970).

Non-linear transforms and reproductive rates (rr)

There is another way of looking at the fitness transform that is not traditional, but which has some advantages when dealing with social selection. Let $\beta(z)$ measure the reproductive rate of change caused by the fitness effect of the trait z such that when $\beta(z) = 0$ there is no change in the proportion of individuals with the trait z. If $\beta(z)$ is greater than zero, there is an increase of the proportion of individuals with the trait z, and if $\beta(z)$ is less than zero, there is a decrease in the proportion of individuals with the trait z. If we set $\beta = 1$ when there is a doubling of the proportion and $\beta = -1$ when the proportion is cut in half, then the distribution of z in the next generation is

$$f'(z) = \frac{f(z)\,2^{\beta(z)}}{\int_{-\infty}^{\infty} f(z)\,2^{\beta(z)} dz} \tag{6}$$

β indicates fitness values in a new way, in terms of a **rate of reproduction** (rr-benefit).

The social selection of signaling

The capacity to believe in unverifiable realities can result from the selection of a signaling pattern. Members of a population signal other members. These signals have fitness costs to the signalers and have fitness benefits, or punishments, for the receivers. I will begin by developing a model for the evolution of signals. It assumes the categories and insights of Maynard Smith and Harper (2003). In connection with religion, the signals are costly. They include participation in rituals, and protestations of belief. Let $v(z)$ indicate the rr-value of a signaling behavior z to a receiver. The receiver chooses to return a rr-benefit, $r(z)$, to the sender. The costs of sending a signal $c(z)$ is also measured as a reproductive rate (rr) benefit. Thus the reproductive effect of sending a signal z in a social context is the benefit returned from receivers minus the cost of sending the signal in the first place.

$$\beta(z) = r(z) - c(z) \tag{7}$$

Leaving out other benefits or costs to signaling in this model, the effect that sending a signal z has on the frequency of z behavior in a next generation is expressed by combining Equations 6 and 7 as follows.

$$f'(z) = \frac{f(z)\,2^{r(z)-c(z)}}{\int_{-\infty}^{\infty} f(z)\,2^{r(z)-c(z)} dz} \tag{8}$$

Equation 8 is a basic formula for the social selection of signaling behavior. This formula allows the expression of benefits or punishments. Boyd and Richerson (1992), Henrich et al. (2006); and Fehr and Gächter (2000), among others, have shown that punishments are important reactions in the evolution of altruism and cooperation. Punishments as well as benefits need to be part of the model. Finding solutions to Equation 8 is not easy. One has to know the nature of the r and c functions. Another problem is measuring z. Equation 8 looks simple, but it hides a complex process that is only barely understood in most cases. However, we can whittle down some of the complexity by proceeding logically from Equation 8.

Defining a religious trait

One can more easily work with simpler forms of Equation 8. I will make some assumptions to make Equation 8 into something that can be more easily evaluated. In the signaling models above, $v(z)$ measures the value of the behavior to a receiver, and $r(z)$ measures the benefits returned to the sender of the signal. To see what happens in a simple model, let us assume a linear relationship between what a receiver receives and what is returned. The final rr-benefit returned to the sender is expressed as.

$$r(z) = a_1 + a_2\, v(z)$$

In this model, we are allowing the receiver to return something, a_1 , independent of what is received. With a constant linear relationship between benefits of signals received and benefits returned to signalers, Equation 8 becomes:

$$f'(z) = \frac{f\,(z)\,2^{[a_1 + a_2 v(z) - c(z)]}}{\int_{-\infty}^{\infty} f\,(z)\,2^{[a_1 + a_2 v(z) - c(z)]}} \tag{9}$$

The behavioral variable z also must be defined. z can be a simple discrete variable with two or more states. For example, z could have two values, $z_1$for generous behavior and z_2 for selfish behavior. z could also be a continuous variable. For example, the value of z could indicate the probability of an individual exhibiting generous behavior. A value of 0 would indicate generous behavior zero percent of the time and 1 would indicate generous behavior 100 percent of the time. When modeling signaling, such as that found in religions, we can measure the signaling behavior by the rr-fitness value it has for a receiver. It could range from some negative number in the punishment (deception) area to some positive number (good information) in the benefits area.

To explore the evolution of religious signaling, consider this type of measure. If the signaling behavior is measured by its value to a receiver, then

$$v(z) = z$$

Another assumption that can be made is that the cost of sending a signal is proportional to its value to a receiver:

$$c(z) = c_1 z$$

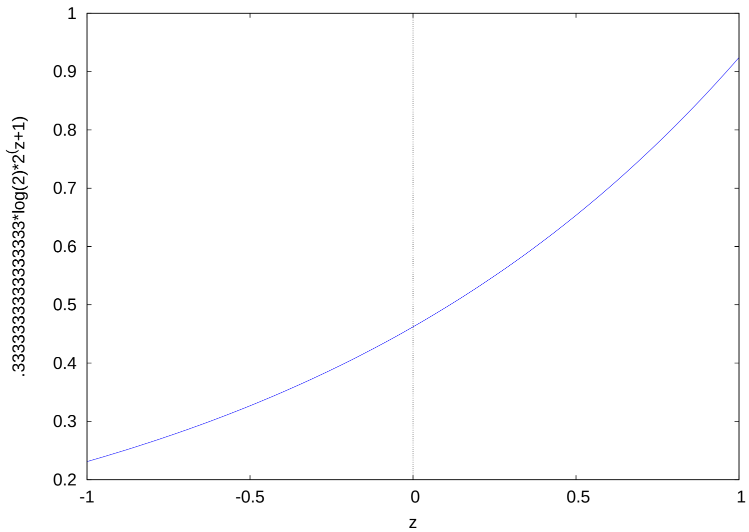


Figure 3: Distribution of behavior in second generation resulting from simple linear return function.

So, Equation 9 now becomes:

$$f'(z) = \frac{f(z)\, 2^{[a_1 + a_2 z - c_1 z]}}{\int_{-\infty}^{\infty} f(z)\, 2^{[a_1 + a_2 z - c_1 z]}}$$

This shows a set of evolving exponential distributions. Plugging in some numbers gives a picture of how this distribution changes. In regards to the evolution of religion, negative values of z represent the signaling of untrue and deceptive information and positive values represent useful signals that benefit the receiver. Figure 3 shows the distribution of z (measured now by its value to receiver) in a second generation when the first generation starts with a uniform distribution of $z = \frac{1}{2}$ between -1 and 1, and assuming $a_1 = 1$, $a_2 = 1$, and $c_1 = 0$. The process skews the evolution toward favoring the high value signalers without end. No equilibrium can be reached.

Our conclusion is that simple linear assumptions such as these do not allow the evolution of religious signaling. For religion to evolve there has to be a process that selects unverifiable information representing the mythological beliefs of the religious group. Even if this information has no or negative reproductive value ($v(z) < 0$), it may be selected if it evokes a return. For example if, $c_1 = 0$, and we use the absolute value of z ,

$$r(z) = 1 + |z|$$

There is a positive return when z is negative, then a second generation of a uniform distribution of z looks like Figure 4.

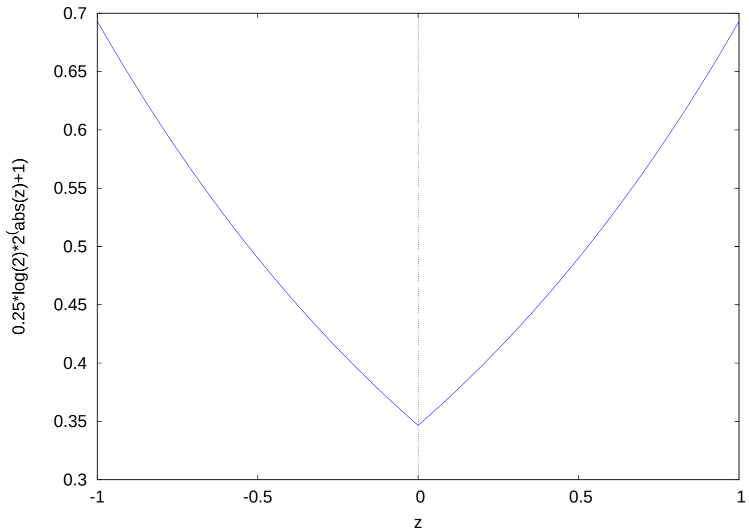


Figure 4: Distribution of z in the second generation when there is a positive return for negative z

This transformation will allow the evolution of behavior with a negative z, the sending of signals that have no positive reproductive benefits to the receiver.

Multiagent Simulation

Because the functions in Equation 8 describing the evolution of signaling are not easy to work with, it is possible that another approach will help to understand them. One approach to the theory of complex systems that has some promise is multiagent simulation. It examines the evolution of complex systems by means of computer simulation. A special approach in multiagent modeling that is particularly applicable to religion is the use of tags (Zohar and Rosenschein 2005, Edmonds 2006). Tags are signals that affect social selection. So far these techniques have not been used to deal with the evolution of religion.

Conclusion

The mathematical simulation of the evolution of signaling patterns holds promise in the understanding of the evolution of religion. The broad outline of a selection process can be expressed mathematically with Lebesgue measure. The equations clearly express the situation. The simplest linear forms of these relationships do not lead toward the evolution of unreal signaling. More complex forms of social selection need to be hypothesized and tested. Beside pure mathematical approaches, multiagent simulation may provide understandable models of the still rather mysterious selection process.

Appendix: Answering questions

Why do the *evogod* graphs start out with level mean values?

The simulation involves cultural learning. The genetic influences are modeled as capacities to learn, not as fixed behaviors. The initial population of agents has no experience, and it takes a little while for it to acquire the behavior.

Why do the real and unreal communications in the *evogod* model start with different values?

Each run involves random initial values and random choices. Only the likelihood of behavior is modeled. The initial genetic tendencies to communicate real and religious information are assigned to the agents at random; therefore, the mean starting values vary slightly from run to run.

Why is $\int_{-\infty}^{\infty} z\, f(z)\, b(z)\, dz - \bar{b}\,\bar{z}$ the same as the "covariance" between z and b

Covariance is the first order central moment of the product of two random variates.

$$\mathrm{Cov}(b,z) = \int_{-\infty}^{\infty} (z-\bar{z})\,(b(z)-\bar{b})\, f(z)\, dz$$

so

$$\mathrm{Cov}(b,z) = \int_{-\infty}^{\infty} \left(-b\,(z)\ f\,(z)\ \bar{z} + \bar{b}\, f\,(z)\ \bar{z} + z\, b\,(z)\ f\,(z) - \bar{b}\, z\, f\,(z)\right) dz$$

Perform the integration on each of the terms to get

$$\mathrm{Cov}(b,z) = -\bar{b}\,\bar{z} + \bar{b}\,\bar{z} + \int_{-\infty}^{\infty} z\, b\,(z)\ f\,(z)\ dz - \bar{b}\,\bar{z}$$

$$\mathrm{Cov}(b,z) = \int_{-\infty}^{\infty} z\, b\,(z)\ f\,(z)\ dz - \bar{b}\,\bar{z}$$

This is the same as the right hand side of Equation 4. Although it is common in population genetics to refer to this as covariance, it should be noted that it is not actually a "covariance," in a purely statistical sense, because $b(z)$ is not a random variate. It is a fitness transform.

References

Candace Alcorta and Richard Sosis. Ritual, emotion, and sacred symbols: The evolution of religion as an adaptive complex. *Human Nature*, 16:323–359, 2005.

Robert Axelrod. *The Evolution of Cooperation*. Basic Books, 1984.

Robert Boyd and Peter J. Richerson. Punishment allows the evolution of cooperation (or anything else) in sizable groups. *Ethology and Sociobiology*, 13: 171–195, 1992.

Pascal Boyer. *The Naturalness of Religious Ideas*. Berkeley: University of California Press, 1994.

Joseph Bulbulia. Religious costs as adaptations that signal altruistic intention. *Evolution and Cognition*, 10:19–42, 2004.

Joseph Bulbulia, Richard Sosis, Erica Harris, Russell Genet, Cheryl Genet, and Karen Wyman, editors. *The Evolution of Religion: Studies, Theories, and Critiques*, 2008. Santa Margarita, CA: Collins Foundation Press.

Michael D. Cohen, Rick L. Riolo, and Robert Axelrod. The emergence of social organization in the prisoner's dilemma: How context-preservation and other factors promote cooperation. Working papers, Santa Fe Institute, Jan 1999. URL `http://ideas.repec.org/p/wop/safiwp/99-01-002.html`.

Harald Cramér. *Mathematical Methods of Statistics*. Princeton University Press, 1945.

Charles Darwin. *The Descent of Man and Selection in Relation to Sex*. London: John Murray, 1871. URL `http://darwin-online.org.uk/content/frameset?itemID=F937.1&viewtype=side&pageseq=1`.

Richard Dawkins. *The Selfish Gene*. Oxford University Press, 1976.

James W. Dow. Is religion an evolutionary adaptation? *Journal of Artificial Societies and Social Simulation*, 11(2), 2008. URL `http://jasss.soc.surrey.ac.uk/11/2/2.html`.

William Durham. *Coevolution: Genes, Culture, and Human Diversity*. Stanford, CA: Stanford University Press, 1991.

Émile Durkheim. *Les Formes Élémentaires de la Vie Religieuse; Le système totémique en Australie*. Les Presses universitaires de France, cinquième edition, 1912.

Bruce Edmonds. The emergence of symbiotic groups resulting from skill-differentiation and tags. *Journal of Artificial Societies and Social Simulation*, 9(1), 2006.

Ernst Fehr and Simon Gächter. Cooperation and punishment in public goods experiments. *American Economic Review*, 90(4):980–994, 2000.

Steven A. Frank. *Foundations of Social Evolution*. Princeton University Press, 1998.

J Henrich, R McElreath, A Barr, J Ensminger, C Barrett, A Bolyanatz, J C Cardenas, M Gurven, E Gwako, N Henrich, C Lesorogol, F Marlowe, D Tracer, and J Ziker. Costly punishment across human societies. *Science*, 312(5781): 1767–1770, 2006.

Laurence R Iannaccone. Introduction to the economics of religion. *Journal of Economic Literature*, 36:273–285, 1998.

William Irons. Religion as a hard-to-fake sign of commitment. In Randolph Nesse, editor, *Evolution and the Capacity for Commitment*, chapter Religion as a hard-to-fake sign of commitment, pages 292–309. New York: Russell Sage Foundation, 2001.

Lawrence Kuznar. *Reclaiming a Scientific Anthropology*. Altimira Press, 1997.

C. C. Li. Fundamental theorem of natural selection. *Nature*, 214:505–506, 1967.

Bronislaw Malinowski. *Magic, Science, and Religion*. Garden City, N. Y.: Doubleday, 1948.

G. E. Marcus and M. J. Fischer. *Anthropology as CulturalCritique: An Experimental Moment in the Human Sciences*. Chicago:Chicago University Press, 1986.

John Maynard Smith. *Evoution and the Theory of Games*. Cambridge University Press, 1982.

John Maynard Smith and David Harper. *Animal Signals*. Oxford: Oxford University Press, 2003.

Richard McElreath and Robert Boyd. *Mathematical Models of Social Evolution: A Guide for the Perplexed*. Chicago: University of Chicago Press, 2007.

G. R. Price. Selection and covariance. *Nature*, pages 520–521, 1970.

A. Robertson. A mathematical model of the culling process in dairy cattle. *Animal Production*, 8:95–108, 1966.

Richard Sosis. Why aren't we all hutterites? costly signaling theory and religious behavior. *Human Nature*, 14:91–127, 2003.

Richard Sosis. The adaptive value of religious ritual. *American Scientist*, 92(2): 166–172, 2004.

Rodney Stark and Roger Finke. *Acts of Faith: Explaining the Human Side of Religion*. Berkeley: University of California Press., 2000.

Edward Burnett Tylor. *Anthropology*. University of Michigan Press, 1960.

Amotz Zahavi. Mate selection — a selection for a handicap. *Journal of Theoretical Biology*, 53(1):205–214, 1975.

Aviv Zohar and Jeffrey S. Rosenschein. Using tags to evolve trust and cooperation between groups, 2005.